Algebra Order of Operation http://www.studygs.net/pemdas

Parenthesis | Exponents | Multiplication | Division | Addition | Subtraction

1. Perform the operations inside a parenthesis first
2. Then exponents
3. Then multiplication and division, from left to right
4. Then addition and subtraction, from left to right
5. You can also create a little phrase to memorize, as the sequence:
   Please Excuse My Dear Aunt Sally

Some Basic Algebra Rules

found at http://www.themathpage.com/alg/rules-of-algebra.htm

$1 \cdot a = a$

$(-1)a = -a \qquad -a = (-1)a$

$-(-a) = a.$

$a + (-b) = a - b. \qquad a - (-b) = a + b.$

$a + b = b + a$

$a + b - c + d = b + d + a - c = -c + a + d + b$

$p - q = p + (-q) = -q + p$

$a \cdot b = b \cdot a$

$abcd = dbac = cdba$

$a + 0 = 0 + a = a$

$a + (-a) = (-a) + a = 0$

If $a = b$, then $b = a$

If $a = b$, then $a + c = b + c$

If $a = b$, then $ca = cb$

If $abx = ac$ then $bx = c$

$a + (b - c + d) = a + b - c + d$

$a - (b - c + d) = a - b + c - d$

$$a \cdot \frac{1}{a} = 1$$

Algebra Basic Properties and Facts

Arithmetic Operations

$$ab + ac = a(b + c)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc} \qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, \quad a \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m} \qquad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$(a^n)^m = a^{nm} \qquad a^0 = 1, \quad a \neq 0$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \qquad \sqrt[n]{\left(\frac{a}{b}\right)} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

Properties of Inequalities

If $a < b$ **then** $a + c < b + c$ **and** $a - c < b - c$

If $a < b$ **and** $c > 0$ **then** $ac < bc$ **and** $\frac{a}{c} < \frac{b}{c}$

If $a < b$ **and** $c < 0$ **then** $ac > bc$ **and** $\frac{a}{c} > \frac{b}{c}$

Distance Formula

If $P_1 = (x_1, y_1)$ **and** $P_2 = (x_2, y_2)$ **are two points, the distance between them is**

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt[n]{a^n} = a$, **if** n **is odd**

$\sqrt[n]{a^n} = |a|$, **if** n **is even**

Factoring and Solving

Factoring Formulas

$$x^2 - a^2 = (x+a)\times(x-a)$$
$$x^2 + 2ax + a^2 = (x+a)^2$$
$$x^2 - 2ax + a^2 = (x-a)^2$$
$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

Square Root Property

If $x^2 = p$ **then** $x = \pm\sqrt{p}$

Quadratic Formula

Solve $ax^2 + bx + c = 0, a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ **- Two real unequal solutions**

If $b^2 - 4ac = 0$ **- Repeated real solutions**

If $b^2 - 4ac < 0$ **- Two complex solutions**

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of ***x*****, square it and add it to both sides**

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for ***x***

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs

Line/Linear Function

$$y = mx + b$$

Graph is a line with point (0,*b*) and slope m.

Slope

Slope of the line containing the two points (x_1, y_1) **and** (x_2, y_2) **is**

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

Slope – intercept form

The equation of the line with slope ***m*** **and passing through the point** $(0, b)$ **is**

$$y = mx + b$$

Point – Slope form

The equation of the line with slope ***m*** **and passing through the point** (x_1, y_1) **is**

$$y = y_1 + m(x - x_1)$$

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2x^2x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{\cancel{a}+bx}{\cancel{a}} \neq 1 + bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling
$-a(x-1) \neq -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2 + a^2} \neq x + a$	$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general version of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$ $(2x+2)^2 = 4x^2 + 8x + 4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parenthesis!
$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$	$\sqrt{-x^2 + a^2} = (-x^2 + a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$